STRUCTURAL DESIGN - I

12. Design for Shear

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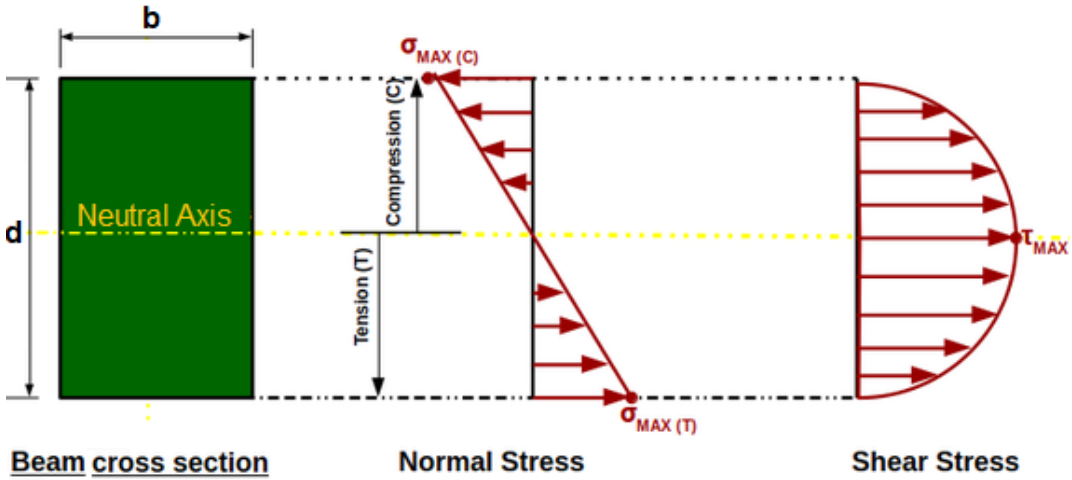
Stresses in Homogenous Rectangular Beams

Bending stress & Shear stress in Beams:

- From basic mechanics of materials, it is known that the flexural (bending) stress f_x and the shear stress τ at any point in the section, located at a distance y from the neutral axis, are given by:

$$f_x = \frac{M \quad y}{I}$$

$$\tau = \frac{V}{I \quad b} \int ydA$$



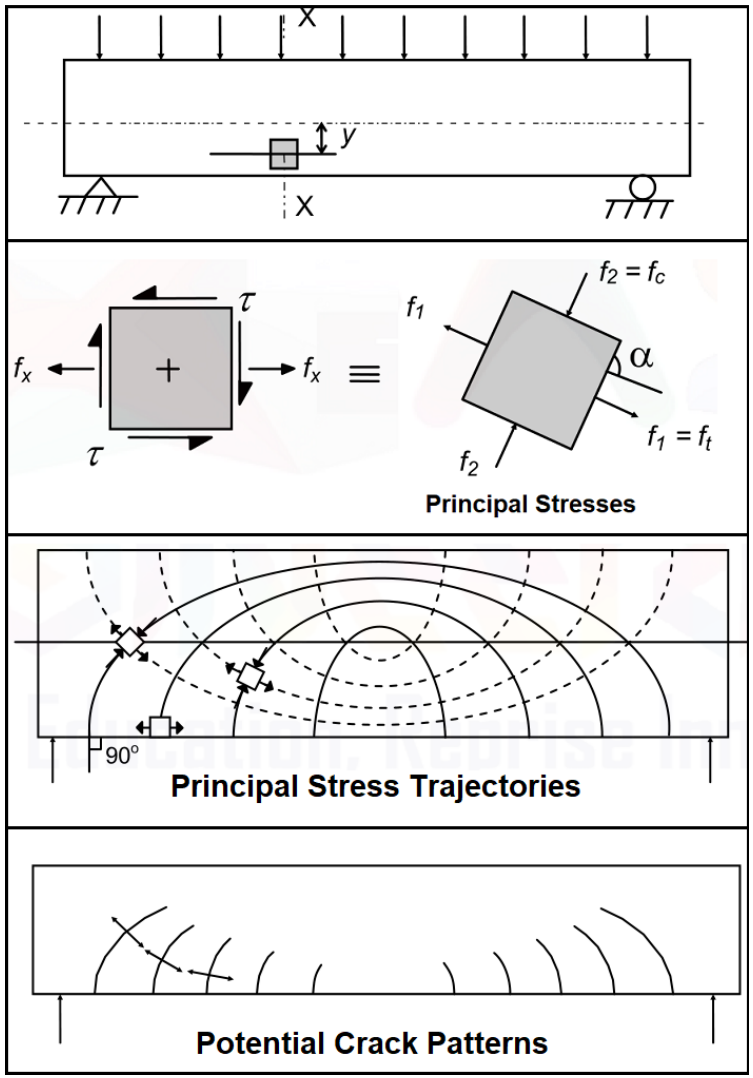
where,

M = Bending moment at any particular section,
V = Shear force at any particular section,
I = second moment of area of the section about the neutral axis,
$\int ydA$ = first moment of area about the Neutral Axis of the portion of the section above the layer at distance y from the NA,
b = width of the beam at the layer at which τ is calculated.

Stresses in Homogenous Rectangular Beams

Principal stresses in beams & Potential crack pattern:

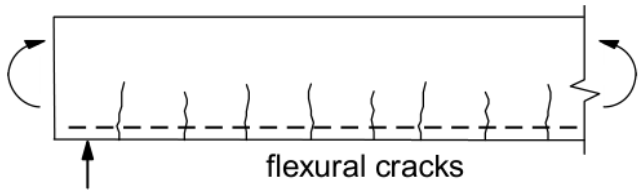
- Any beam element at *'y'* distance from the Neutral Axis experiences normal stress (f_x) & shear stress (τ).
- Combined effect of f_x & τ can be resolved into equivalent principal stresses f_1 and f_2 acting transverse to eachother.
- Note that, f_1 is tensile, while f_2 is compressive in nature. And the stress on the beam is completely expressed in terms of the principal stresses as shown (see Principal stress trajectories).
- Since concrete is weak in tension, tensile cracks would develop perpendicular to f_1 (or in other words, along f_2).
- Thus the compressive stress trajectories in the above figure indicate potential crack pattern.



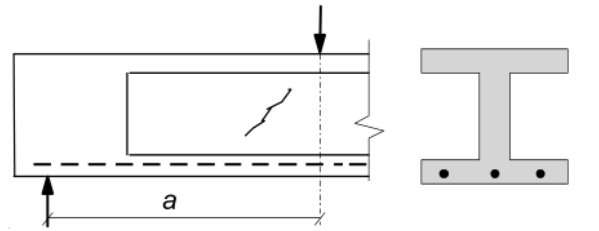
Behaviour of Reinforced concrete under Shear

Modes of Failure/Cracking:

1. **Flexural cracks**
   - Occurs in RC beams, subjected to high bending stress and low shear stress.
   - Forms at 90^o from the extreme tension fibre towards neutral axis. Resisted by tension reinforcements.

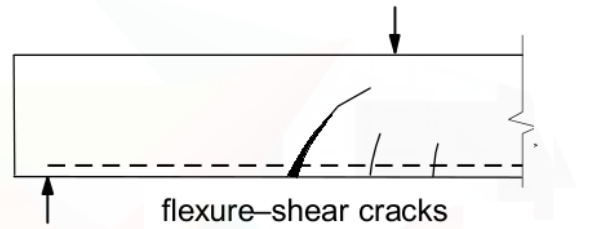


2. **Diagonal Tension Cracks / Web shear cracks**
   - Occurs in beams, subjected to high shear stresses (due to heavy concentrated loads or in deepbeams).
   - Forms near the supports inclined at 45^o to the longitudinal axis of the beam. Resisted by providing shear reinforcements.

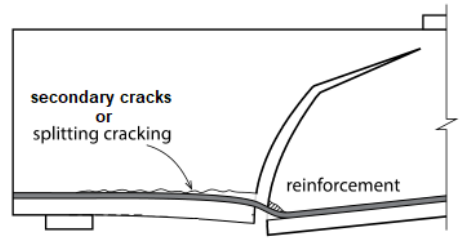


web–shear crack **(or Diagonal Tension Crack)**

3. **Flexure-Shear cracks**
   - Occurs in beam subjected to both flexure and shear. Flexural crack occurs in combination with a web-shear crack.



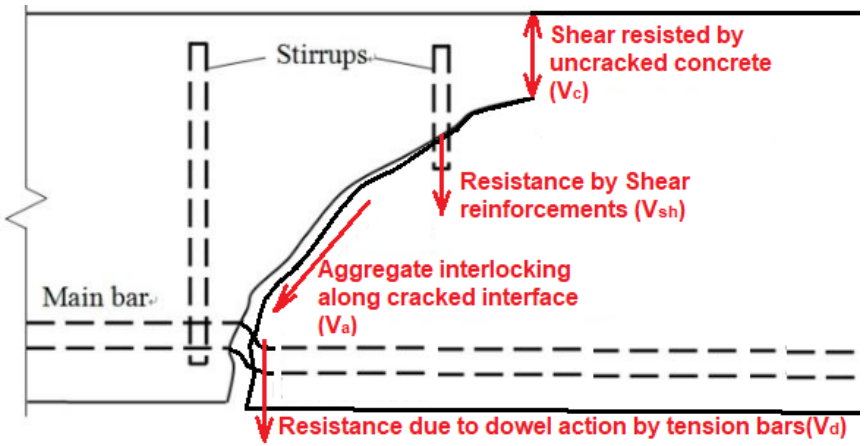
4. **Secondary cracks / Splitting cracks**
   - Flexure-Shear cracks may sometimes propagate along the tension reinforcement towards the support, due to either 'wedging action' or 'dowel action' of tension bar.



Behaviour of Reinforced concrete under Shear

Shear Transfer mechanism:

- **The following internal forces develop to resist shear:**
  1. Shear resistance by uncracked portion of concrete (V_c).
  2. Shear resistance by aggregate interlocking, along crack interface (V_a).
  3. Dowel action by tension bar (V_d)(steel bar resisting relative dispacements between the beam segments on either sides of the crack is called *Dowel Action*)
  4. Shear resistance by Shear reinforcement(V_{sh}).



- **Sequence of events in shear transfer:**
  1. Prior to initiation of crack, shear is resisted entirely by V_c.
  2. As flexural crack initiates, some shear is resisted by V_a and V_d, in addition to V_c.
  3. As diagonal tension crack initiates , shear reinforcement comes now to offer resistance (V_{sh}), by intercepting the crack and yields as loading increases. Hence, shear reinforcements ensures that the shear failure is **gradual** and **ductile**.

Shear parameters for Design

Nominal Shear Stress (τ_v) :

40 LIMIT STATE OF COLLAPSE : SHEAR

40.1 Nominal Shear Stress Page 72

The nominal shear stress in beams of uniform depth shall be obtained by the following equation:

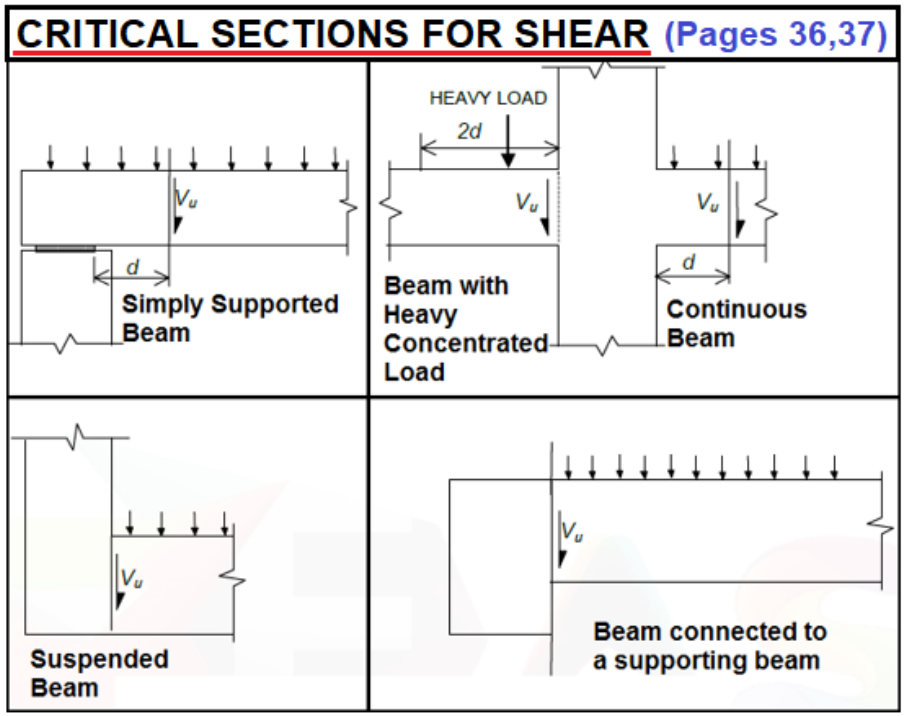
$$\tau_v = \frac{V_u}{bd}$$

where

V_u = shear force due to design loads;

b = breadth of the member, which for flanged section shall be taken as the breadth of the web, b_w; and

d = effective depth.



Shear parameters for Design

Design Shear strength of concrete (τ_c) :

- It is the limiting maximum shear stress the RC beam resists *without shear reinforcement*.
- If $\tau_v < \tau_c$, shear reinforcements neednot be designed (only minimum shear reinforcements shall be provided).
  But if $\tau_v > \tau_c$, shear reinforcements must be designed.

40.2 Design Shear Strength of Concrete

40.2.1 The design shear strength of concrete in beams without shear reinforcement is given in Table 19.

40.2.1.1 For solid slabs, the design shear strength for concrete shall be $\tau_c k$, where k has the values given below:

Overall Depth of Slab, mm	300 or more	275	250	225	200	175	150 or less
k	1.00	1.05	1.10	1.15	1.20	1.25	1.30

NOTE — This provision shall not apply to flat slabs for which **31.6** shall apply.

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Table 19 Design Shear Strength of Concrete, τ_c, N/mm²
(*Clauses* 40.2.1, 40.2.2, 40.3, 40.4, 40.5.3, 41.3.2, 41.3.3 *and* 41.4.3)

(Page 73)

$100\frac{A_s}{bd}$	Concrete Grade M 15	M 20	M 25	M 30	M 35	M 40 and above
(1)	(2)	(3)	(4)	(5)	(6)	(7)
≤0.15	0.28	0.28	0.29	0.29	0.29	0.30
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.75	0.71	0.82	0.90	0.94	0.96	0.98
3.00 and above	0.71	0.82	0.92	0.96	0.99	1.01

NOTE — The term A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at support where the full area of tension reinforcement may be used provided the detailing conforms to **26.2.2** and **26.2.3**

- **Note:** Shear strength of slabs is higher than that of beams, because of its smaller thickness. Thinner slabs have greater shear strength. As per Cl. 40.2.1.1, τ_c of slabs shall be multipled by a factor k which ranges from 1 to 1.3. **Generally, we find that $\tau_v < k\tau_c$ in slabs, and hence do not need shear reinforcement.**

Shear parameters for Design

Limiting Ultimate Shear Resistance of Beam ($\tau_{c,max}$):

- It is the limiting maximum shear stress the RC beam resists *with shear reinforcement*.
- Such a limit is set to τ_v because :
  - With excessive shear reinforcement in the section, the beam looses its ductile nature.
  - Hence, failure may occur by crushing of concrete, even before yielding of shear reinforcements,which is undesirable. This is known as **Shear-compression failure**.
  - Therefore, Beams must satisfy the condition: $\tau_v < \tau_{c,max}$

Table 20 Maximum Shear Stress, $\tau_{c\,max}$, N/mm²

(Clauses 40.2.3, 40.2.3.1, 40.5.1 and 41.3.1)

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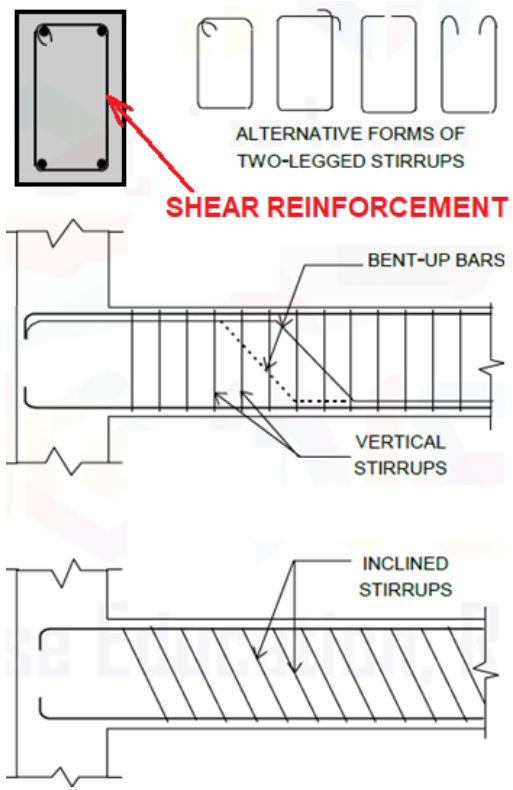
Concrete Grade	M 15	M 20	M 25	M 30	M 35	M 40 and above
$\tau_{c\,max}$, N/mm²	2.5	2.8	3.1	3.5	3.7	4.0

Design of Shear Reinforcements

Types of Shear Reinforcements:

- Shear reinforcements maybe provided in any one of the following ways (Cl. 40.4)
  - Stirrups perpendicular to the beam axis;
  - Stirrups inclined (at 45^0 or more) to the beam axis; and
  - Longitudinal bars bent-up at 45^0 to 60^o to the beam axis, combined with stirrups.
- Most common type is the **two-legged stirrups**, comprising a closed or open loop. It is placed perpendicular to the member axis (vertical stirrup), and may or may not be combined with bent-up bars.
- Where bent-up bars are provided, their contribution towards shear resistance shall not be more than half that of the total shear reinforcement.



Design of Shear Reinforcements

Codal recommendations for Shear Reinforcements:

Based on the relative magnitudes of τ_v and τ_c, shear reinforcements are provided as follows:

- If $\tau_v > \tau_c$,
  1. Provide shear reinforcements, as per Cl. 40.4 (Page73) of IS456
  2. Shear force to be resisted by stirrups

$$V_{us} = V_u \text{ - } \tau_c \text{ b d}$$

where,if vertical stirrups are used,

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

where

A_{sv} = cross-sectional area of stirrup legs or bent-up bars,

$= n(\frac{\pi\phi^2}{4})$, with ϕ the diameter of n-legged stirrups,

s_v = center-to-center spacing of the stirrups along the length of the member,

d = effective depth of beam

  3. From above equation, the required spacing s_v of vertical stirrups can be determined, since the value of all other terms are known.
  4. For vertical stirrups, the maximum spacing between stirrups should not exceed 0.75d or 300mm, whichever is minimum (Cl. 26.5.1.5 - Page47).

Design of Shear Reinforcements

Codal recommendations for Shear Reinforcements:

- If $\tau_v < \tau_c$,
  1. If $\tau_v < 0.5\tau_c$
     No shear reinforcement is required at all.

  2. If $\tau_v > 0.5\tau_c$
     - The Code (Cl. 26.5.1.6 - Page48) specifies a minimum shear reinforcement to be provided in the form of stirrups.

$$\frac{A_{sv}}{bs_v} \geqq \frac{0.4}{0.87f_y}$$

     - From above equation, the required spacing s_v of vertical stirrups can be determined, since the value of all other terms are known.
     - For vertical stirrups, the maximum spacing between stirrups should not exceed 0.75d or 300mm, whichever is minimum (Cl. 26.5.1.5 - Page47).

Now solve numerical examples!!!